Lecture 15

3rd Semester M Tech. Mechanical Systems Design

Mechanical Engineering Department

Subject: Advanced Engine Design

I/C Prof M Marouf Wani

Lecture 15 – Balancing of Mechanical Forces in I C Engines
Topic: Balancing of Reciprocating Forces in Internal Combustion Engines - 20-10-2020

The piston, piston pin, rings, and the upper portion of the connecting rod are subject to **reciprocating motion** and thus **repeated acceleration and deceleration.**
The **reciprocating forces act along** each cylinder **centerline**, and their **magnitude varies** continuously **with crank angle**.
The magnitude of the reciprocating forces at any given crank angle position can be determined from **Newton's Second Law** as the product of the reciprocating mass and the instantaneous acceleration at that crank angle.
Piston velocity reaches zero at both the TC and BC positions, following a non symmetric trace determined by the slider-crank geometry between those positions.
The peak velocity does not occur midway between TC and BC, but is skewed **toward TC**.

If the connecting rod were infinitely long, the velocity trace would be sinusoidal, but as the connecting rod is made shorter, the velocity trace becomes increasingly skewed.

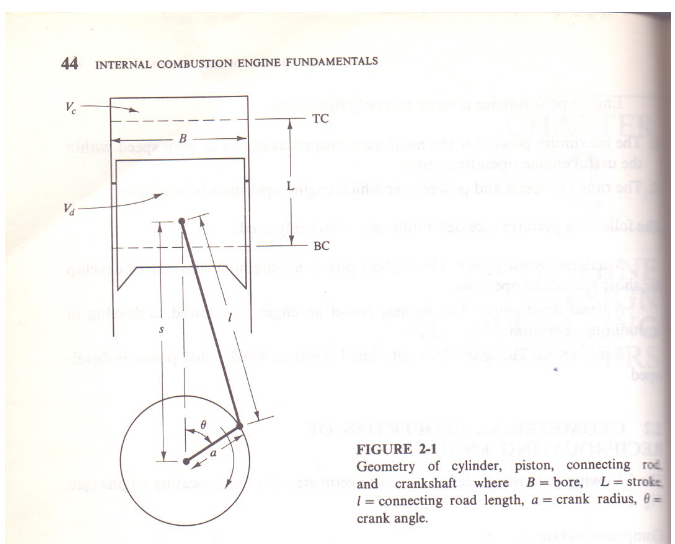

FIGURE 2-1
Geometry of cylinder, piston, connecting rod and crankshaft where B = bore, L = stroke, l = connecting road length, a = crank radius, θ = crank angle.

The expression for the distance of piston pin from the crankshaft centre given in the book by John b Heywood is

$s = a\cos\Theta + (l^2 + a^2 sin^2\Theta)^{1/2}$

By double differentiation of this equation on both sides:
First we get
The equation for the velocity of the piston
Second we get
The equation for the acceleration of the piston

The resulting velocity versus crank angle **series expression** given in the book by Kevin L Hoag is of the type:

$Sp = -\omega^2 a[\sin\Theta + 2a_2\sin 2\Theta + 4a_4\sin 4\Theta + -----]$ ---------------------------- 1

Where
ω = Crankshaft speed in radians per unit time.
a = radial position of the rod bearing centre versus crankshaft centre (stroke/2)
or
a = crank radius
Θ = crank angle relative to TC

$a_2 = \frac{l}{a}\ [\frac{1}{4}(\frac{a}{l})^2 + \frac{1}{16}(\frac{a}{l})^4 + \frac{15}{512}(\frac{a}{l})^6 + ------]$ ………………………………………..2

$a_4 = \frac{l}{a}\ [\frac{1}{64}(\frac{a}{l})^4 + \frac{3}{256}(\frac{a}{l})^6 + ------]$ ---3

Where

l = length of the connecting rod.

Of current interest, is **the instantaneous acceleration**, which is simply the **velocity derivative with respect to time** (in this case crank angle):

$\frac{dSp}{d\Theta} = -\omega^2 a\ [\ \cos\Theta + 4a_2\cos 2\Theta + 16a_4\cos 4\Theta + -----]$ …………………………….4

The **instantaneous reciprocating force at any given cylinder** can now be calculated as
F = m*a

$F_{reciprocating} = M_{piston\ assy} * \frac{dSp}{d\Theta}$

$F_{reciprocating} = -M_{piston\ assy} * \omega^2 a\ [\ \cos\Theta + 4a_2\cos 2\Theta + 16a_4\cos 4\Theta + ---]$ ……..5

Equation 5 can be greatly simplified by making **two observations**.

First, unless the connecting rods are very short, it is observed that the **higher –order terms are quite small** and only the **first two must be considered**.

Second, the **term $4a_2$** is observed to be **almost** identically $\frac{a}{l}$

$4a_2 = \frac{a}{l}$

These observations result in the simplified acceleration expression

$F_{reciprocating} = - M_{piston\ assy} * \omega^2 a\ [\ \cos\Theta + 4a_2 \cos 2\Theta]$ ……………………………………….6

$F_{reciprocating} = - M_{piston\ assy} * \omega^2 a\ [\ \cos\Theta + \frac{a}{l} \cos 2\Theta]$ …………………………………………….7

From equation 7 it is seen that:

1. The **first crank angle term**, **involving $\cos\Theta$** , gets **repeated once every crank shaft revolution** and can be defined as the **First Order Force** --- defined previously for first order vibration
2. The **second crank angle term, involving $\cos 2\Theta$,** gets **repeated twice every crank shaft revolution** and can be defined as the **Second Order Force** --- defined previously for second order vibration

64 Engine configuration and balance

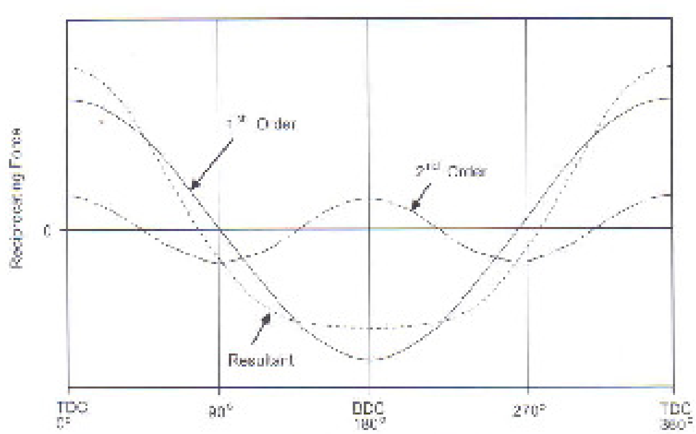

Fig. 6.5. First- and second-order reciprocating forces and the resultant force versus crank angle for an individual cylinder

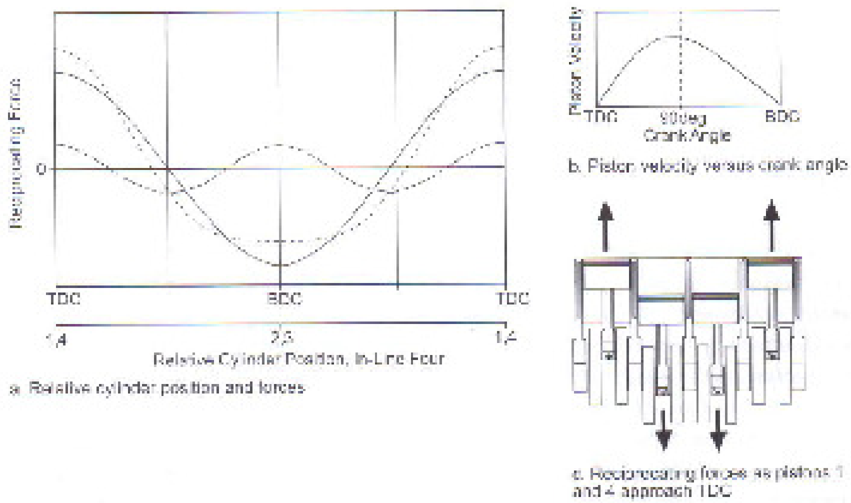

Fig. 6.6. Reciprocating forces in an in-line 4-cylinder engine

Dated: 20-10-2020

Prof M Marouf Wani

I/C Advanced Engine Design
Mechanical Engineering Department
National Institute of Technology
Srinagar, J&K
India – 190006

Text Book:
Vehicular Engine Design
By Kevin L. Hoag
Published By: SAE International USA